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DUAL ORTHOGONAL SERIES AND MIXED BOUNDARY VALUE PROBLEMS.(U)  
JUL 77 R B KELMAN

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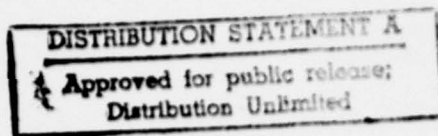


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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>In this project efforts were made at developing mathematical theory and computer programs for the study and use of dual orthogonal series of the form summation of $j_n((a)_n P_n + (b)_n Q_n) = f$ . Both the abstract space theory of such series and the analytic properties of special cases were studied. |  |   |



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## Final Report

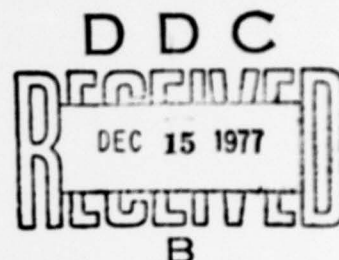
Dual Orthogonal Series and Mixed Boundary Value Problems

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In this project efforts were made at developing mathematical theory and computer programs for the study and use of dual orthogonal series of the form

$$(1) \quad \sum j_n (a_n P Q_n + b_n Q d_n) = f.$$

Here  $\{\phi_n\}$ , the kernel is a complete orthonormal system for a Hilbert space  $R$  which is the direct product of the orthogonal subspaces  $P$  and  $Q$ .  $f$  is a given element of  $R$ ,  $\{a_n\}$  and  $\{b_n\}$  are given sequences of real constants, and  $P$  and  $Q$  are the projection operators from  $R$  onto  $P$  and  $Q$  respectively. We seek  $\{j_n\}$ .

We have studied both the abstract space theory of such series and the analytic properties of special cases. The classic dual series problem consists of the case in which  $R = L^2(0, \pi)$  and  $P = (0, b)$  for some positive constant  $b$  less than  $\pi$ , and the kernel has one of the trigonometric forms  $\{\sin nx\}$ ,  $\{\cos nx\}$ ,  $\{\cos(n-1/2)x\}$ , or  $\{\sin(n-1/2)x\}$ . In addition  $a_n = 1$ , and  $b_n$  has one of the forms  $1/(n+h)$  or  $1/(n+1/2+h)$  where  $h$  is a nonnegative constant. In [A1] the case  $h = 0$  was investigated and a thorough analysis established rigorously the conditions under which existence and uniqueness could be guaranteed and provided criteria for the pointwise convergence of the series in the sense of Abel-Poisson. Perhaps the most important finding was showing that in general the series do not converge on any set of positive measure in the ordinary sense. These results were then extended to the classical triple trigonometric series in [A4] wherein it was pointed out that for combinatorial reasons the method of proof used in [A1, A4] could not be extended to  $n$ -part classic trigonometric series problems for  $n > 4$ . In [A5], the results were established for the case  $h > 0$ . As pointed out in [A5] there have been relatively few analyses considered for such series. Even though the theoretical results are similar to those found in [A1],

there are considerable technical difficulties associated with proving and developing the formulas associated with  $h > 0$  because of that development in the conformal mapping techniques employed in [A1] and [A4].

In dealing with these classical series many specific formulas have been developed since the 1960's to provide solutions in terms of double singular integrals. These formulas are analytically useful, but in their present state of development have required further analysis for two reasons: (i) they are not algorithmic and (ii) their range of validity has never been rigorously established. In [A3] and [A6] these formulas have been converted into theoretical algorithms with computer implementation. As a result the ability to use them has been greatly enhanced in applications such as fracture mechanics. Further, the algorithmic formulas have provided a key to rigorously analyzing such equations and some results in this direction were given in [R2].

A further programming effort to simplify the numerical solution of general dual trigonometric series is given in [R1]. This programming package provides the user with the ability to solve quite general dual trigonometric series by the use of at most three FORTRAN statements.

A short review article indicating the principal findings for the abstract version of (1) was published [A2] to give wider exposure to the subject of dual series.

A new application of dual series to the very active area of cryptology was developed, and the basic theoretical results on the ability to construct a decoder will be published [A7].

At the present time we are writing a paper for the purpose of exposing some of the ideas in dual orthogonal series to a wider audience. [P1] will be submitted to the section on "Classroom Notes in Applied Mathematics" of SIAM Rev. and will deal with didactics of introducing discontinuous boundary value problems into a first course in applied mathematics. Illustrative examples will be chosen from current problems in technology, e.g., minimal size of computer elements, transportation of liquified natural gas, and heat transfer in the presence of imperfections.

[P2] represents the completion of the work begun in [A6]. Formulas for the classic dual cosine series tend to have distinct characteristics because of their association with Neumann conditions. The algorithmic description for this equation although similar to [A6] requires a new analysis and hence a new programming package. With the completion of [P2] algorithms will be available for all classic dual trigonometric equations.



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- R2. J. T. Simpson, Existence and uniqueness theorems for dual trigonometric series, Ph.D. Thesis, Dept. of Mathematics, Colorado State University, Ft. Collins, June 1975.

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- D1. J. T. Simpson, Ph.D. in Mathematics, Colorado State University, August, 1975.

### Papers in Preparation

- P1. R. B. Kelman, Least squares Fourier series solutions to boundary value problems.
- P2. R. B. Kelman, Algorithmic analysis for the classic dual cosine equation.

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